Acoustical influence of the geometry of a clarinet Bb reed by means of a parametric analysis

Iñigo Martínez [a] , Jesus Paredes [a] , Maite Gárate [a] , Coro Celigüeta [a]
[a] Tecnun-University of Navarra, Manuel de Lardizabal 13, 20018 San Sebastián, Spain

Abstract

The reed is one of the most important elements that form a clarinet, and its geometry is the key to the correct production of the sound. This study pretends to investigate the influence of the geometry of the reed in the acoustics of the clarinet. First a theoretical model (Four-Pole Method) is used to study the acoustic performance of the clarinet. Then, an acoustic model has been created and a parametric modal analysis of the reed has been carried out. The obtained simulations have been validated by means of experimental data from a real Bb clarinet. Moreover, an harmonicity study was performed out the experimental data.

La lengüeta es uno de los elementos más importantes del clarinete, y su geometría es crucial para la correcta producción del sonido. Este trabajo pretende comprender la influencia de la geometría de la lengüeta en la acústica del clarinete. Se ha partido de un modelo teórico (Four-Pole Method) para estudiar el rendimiento acústico del clarinete. Posteriormente se ha modelado acústicamente el clarinete y se ha realizado un análisis modal paramétrico de la lengüeta. Las simulaciones realizadas se han validado por medio de datos experimentales con un clarinete Bb real, del que se ha realizado un estudio de armonicidad.

***Key Works**: Acoustics, Vibrations, Clarinet, Reed, Natural Frequency, Harmonicity*

I. INTRODUCTION

This research attempts to understand the mode of operation of the clarinet, to obtain the waveform of the clarinet and study the reed from the point of view of vibrations.

The clarinet is a woodwind instrument that uses a reed to induce vibrations in the air column and make sound. The reed is made out of an elastic material called Arundo Donax, a variety of cane. This reed is attached over the window of the mouthpiece where the clarinetist blows air into the system. As the clarinet is played, a pressure antinode forms in the mouthpiece, which makes the clarinet similar to a pipe closed at one end and open at the other. [1]

The B-flat clarinet is about 60 cm long and has a range of more than three octaves. Different notes containing several harmonics are produced by opening and closing various tone holes along the bore of the instrument.

This work will only focus on the production of one musical note, specifically on E3 (Mi - 147 Hz). This note corresponds with the lowest note that it is possible to play on a Bb clarinet.

Over the past years several studies about windwood instruments, in particular about the clarinet, have been developed, mostly focused on the modeling of the clarinet's reed.

The stated mathematical models for the modeling of the reed date from around the 1960s, for instance, the model developed by Backus [2]. However, the results of these models were limited until the development of the computer systems. In the 1980s, the first computer models were implemented, as the ones made by Stephen E. Stewart and William J. Strong. These studies [3] considered a simplified clarinet by approaching to a electric power transmission line and its equivalent circuit. Among the obtained results, it is possible to find the relationship between the pressure and the air flow as well as the frequency range of the reed.

Picard et al. (Analysis of Clarinet Reed Oscillations With Digital Fresnel Holography [4]) focuses on the experimental measurement of the reed's vibrations, but a reed model is not formed. The test method is based in two ways to excite the reed. On the one hand, the excitement is produced by an acoustic controlled frequency wave. On the other hand, the excitement is produced by an artificial mouth. The deformations of the reed are digitally measured by using digital Fresnel holograms, and the reed's natural frequencies are obtained from these deformations.

Other studies, as the one realized by Vasileios Chatziioannou and Maarten van Walstijn [5], focus on the modeling of the reed considering its anisotropy by using a two-dimensional model and its numerical resolution. This study takes into account the effects of modifying the reed shape and the position of the player's lips. However, has some restrictions: it does not consider the effects caused by the non-linearity as well as an experimental model.

Some other studies as the ones by P.Taillard [6] et al. concentrates in the estimation of parameters that define the model of the reed. The holographic system was also used in this study. The measured parameters were analyzed statistically and compared with the numerical results. The final conclusion states that the numerical models are not good enough.

So far, the effect of the reed geometry on the sound as well as on the tone of the clarinet has not been studied. Therefore, this topic will be addressed through this work. First, the acoustics in the tube of the clarinet and the influence of the geometry of the reed in its own natural frequency are analyzed. Subsequently, it is verified that a change in natural frequency of the reed linearly affects the frequency perceived by the air, and therefore the note played by the clarinetist.

Therefore, the objectives of this research can be summarized as follows:

- Related to the acoustic in the pipe:
  - To obtain an acoustic model of a clarinet by using the commercial software ANSYS. The study begins with a open-closed end pipe, based on the actual dimensions of a "Conn-Selmer Prelude Student Model CL711 Bb Clarinet"
  - To analyze the natural frequencies and the vibration modes of the air inside the pipe, theoretically (by ANSYS) and experimentally (signal acquisition by MATLAB).
  - To compare the natural frequencies obtained both theoretically and experimentally.
- Related to the reed geometry:
  - To obtain the key geometrical parameters of the reed.
  - To analyze the natural frequencies and the vibration modes of the reed.
  - To study the influence of the geometric parameters in the natural frequencies of the reed, by means of a sensitivity analysis.
  - To compare the frequency response, first, of the reed isolated from the rest of the system, and second, of the set reed-tube.

Consequently, this study will provide data to understand the geometry of the reed.

This article is structured as follows:

This chapter introduces the acoustics of the clarinet and carries out a review of the research works developed so far. In addition, the objectives of this project are described. In chapter 2, the methods section, the applied theory used to perform this work is presented: acoustic theory, modal analysis, harmonic response, the four-pole transmission line method, etc. This chapter also includes the designed models of the pipe and the reed as well as the process followed in ANSYS. Chapter 3 presents and discusses the results obtained for the pipe and the reed. Conclusions and future works are presented in Chapter 4.

II. Methodology

A. Cylindrical open-closed pipe model

The understanding of a cylindrical open-closed pipe model becomes necessary to understand the modal analysis and the frequency response studied in this work. A pipe needs to fulfil the following two requirements to perform as a woodwind instrument:

- On the one hand, the stationary acoustic waves formed in the pipe must keep a fixed relationship between the vibrations modes frequencies of the air column located in the pipe.

- On the other hand, the highest vibration modes frequencies must be approximately an integer multiple of the fundamental vibration mode. This restriction largely restricts the different pipe shapes that can be used to manufacture the woodwind instruments. As this work focuses on the clarinet, a cylindrical pipe analysis is adopted as an efficient approach, where the symmetry axis corresponds with the X coordinate axis, as shown in Fig. 1.

Considering the equation of the motion of a planar wave in the x direction, taking into account both directions [7]:

$$\frac{1}{c^2} \cdot \frac{\partial^2 P}{\partial t^2} = \frac{\partial^2 P}{\partial x^2} \qquad (1)$$

The general solution for this equation is well known:

$$P(x,t) = (Ae^{-ikt} + Be^{ikt})e^{-i\omega t} \qquad (2)$$

where $k = w/c_0$ is the wavenumber, $w = 2\pi f$ is the circular frequency, f is the frequency of excitation, and c_0 is the speed of sound. The volume of acoustic flux per unit of area as:

$$U(x,t) = \left(\frac{S}{\rho c_0}\right)(Ae^{-ikt} - Be^{ikt})e^{-i\omega t} \qquad (3)$$

Where S is the cross-sectional area, and ρ is the density. With these magnitudes, the acoustic impedance can be defined as:

$$Z(x) = \frac{P(x)}{U(x)} \qquad (4)$$

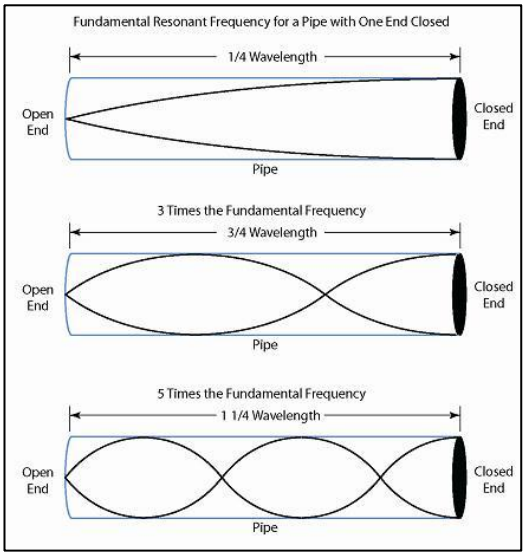


Fig. 1. Resonance pressure in a pipe with one end closed: 1st, 3rd ,5th modes

Only certain modes of pressure and longitudinal displacement waves occur in various air column configurations [1]. Since the clarinet is similar to a cylindrical pipe with one end open and one end closed, an open-closed pipe, the focus for this section will be on this air column configuration. Fig. 1 depicts the first three modes of displacement and pressure waves within an open-closed pipe. The closed end allows for a displacement node and a pressure antinode while the open end allows the opposite. The distance between a node and antinode is a quarter wavelength, so that

$$\lambda_1 = 4L \tag{5}$$

where λ_1 is the wavelength for the first wave mode and L is the length of the pipe. The fundamental frequency then becomes,

$$f_1 = \frac{c_0}{4L} \tag{6}$$

Using wavelengths and pipe lengths to calculate subsequent mode frequencies in terms of the fundamental frequency, a pattern develops. The wavelengths become

$$\lambda_1 = \frac{4L}{n} \tag{7}$$

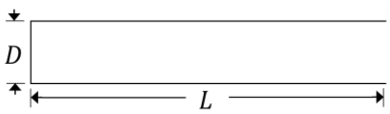
and the corresponding frequencies are

$$f_n = n\, f_1 \tag{8}$$

where n is an odd integer. Therefore, only frequencies that are odd integer multiples of the fundamental are produced in an open-closed end pipe [1]. The first mode is referred to as the fundamental or first harmonic, which is followed by the third harmonic, fifth harmonic, and so on.

The natural frequencies and mode shapes of undamped tubes are listed in Table 1

TABLE I. NATURAL FREQUENCIES AND AXIAL MODE SHAPES OF OPEN-RIGID TUBE

Configuration	Open-rigid
Schematic	
Mode Index	$n = 1,3,5,7 \ldots$
Natural Frequencies f_n	$f_n = \frac{n\, c_0}{4L}\ [Hz]$
Mode Shape ψ_n	$\psi_n = \cos\left[\frac{n\pi x}{2L}\right]$

B. *Four-Pole Method*

The four-pole is a useful theoretical tool for estimating the acoustic performance of tubes.

An acoustic source, such as the clarinetist air flow, is attached to the tube, that has an acoustic impedance and can be represented by 4-pole transmission line matrix [8, p. 103]. The acoustic source has an impedance Z_s. The end of the acoustic duct has a termination impedance Z_T , which in the tube shown in Fig. 2, is the radiation impedance of an unflanged duct radiating into a free-field.

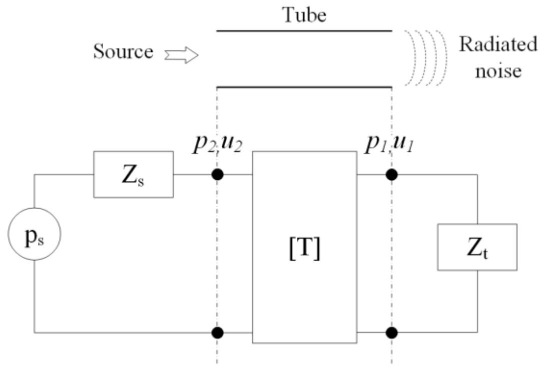


Fig. 2. Schematic of the 4-pole transmission matrix method.

The pressure and mass velocity upstream and downstream of an element are related by a 4-pole transmission matrix as

$$\begin{bmatrix} p_2 \\ \rho_0 S_2 u_2 \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{bmatrix} p_1 \\ \rho_0 S_1 u_1 \end{bmatrix} \tag{9}$$

$$\begin{bmatrix} p_2 \\ V_2 \end{bmatrix} = T \begin{bmatrix} p_1 \\ V_1 \end{bmatrix} \tag{10}$$

where p_i is the acoustic pressure at point i along the system. The mass own velocity V_i is the density of the gas times cross-sectional area of the duct times the acoustic particle velocity and is calculated as

$$V_i = \rho_0 S_i u_i \tag{11}$$

where ρ_0 is the density of the gas, S_i is the cross-sectional area of the duct at point i, and u_i is the acoustic particle velocity (not the mean own velocity) at point i.

The 4-pole transmission matrix for a straight segment of duct of length L is given by [8, p. 104]

$$T = \begin{bmatrix} \cos kL & j\dfrac{c_0}{S}\sin kL \\ j\dfrac{c_0}{S}\sin kL & \cos kL \end{bmatrix} \tag{12}$$

The equations describing the response of the system shown in Fig 2 can be written as

$$\begin{bmatrix} p_s \\ \rho_0 S_s u_s \end{bmatrix} = \begin{bmatrix} 1 & Z_s \\ 0 & 1 \end{bmatrix} \begin{bmatrix} p_2 \\ \rho_0 S_2 u_2 \end{bmatrix} \quad (13)$$

$$\begin{bmatrix} p_2 \\ \rho_0 S_2 u_2 \end{bmatrix} = T \begin{bmatrix} p_1 \\ \rho_0 S_1 u_1 \end{bmatrix} \quad (14)$$

$$\begin{bmatrix} p_1 \\ \rho_0 S_1 u_1 \end{bmatrix} = \begin{bmatrix} 1 & Z_T \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ \rho_0 S_1 u_1 \end{bmatrix} \quad (15)$$

where the 4-pole transmission matrix [T] depends on the configuration of the duct segment. These equations can be written in matrix form as

$$\begin{bmatrix} p_s \\ \rho_0 S_s u_s \end{bmatrix} = \begin{bmatrix} 1 & Z_s \\ 0 & 1 \end{bmatrix} T \begin{bmatrix} 1 & Z_T \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ \rho_0 S_1 u_1 \end{bmatrix} \quad (16)$$

The impedance of an unflanged pipe radiating into a free-field is given by [8, p. 105]

$$Z_T = R_0 + jX_0$$

$$Z_T = \left[\rho_0 c_0 S \frac{(ka^2)}{4} \right] + j[(\rho_0 c_0 S) 0.61\ ka] \quad (17)$$

where a is the radius of the duct at the exit, R_0 is the real part of the impedance called the resistance, and X_0 is the imaginary part of the impedance called the reactance.

C. Fluid-Structure Interaction Using ANSYS

The acoustic-modal analysis is similar to the well known structural-modal analysis. In fact, a modal analysis can be conducted to calculate the natural frequencies and mode shapes of an acoustic or structural system, or a combined structural-acoustic system. It is possible to obtain the frequency response in every system's degree of freedom considering the obtained results in the mode analysis.

The equations of the modal theory are included in the annex A, in case the lector would like to make a review. It includes the notion of natural frequencies and mode shapes.

Given that this works englobes both the acoustics theory and the modal theory, it is essential to explain the mathematical procedure used by ANSYS in order to understand the simulations and obtained results.

The equations of motion for the structure are [8, p.14]

$$[\mathrm{M}_s]\{\ddot{U}\} + [\mathrm{K}_s]\{U\} = \{\mathrm{F}_s\} \quad (18)$$

where $[\mathrm{K}_s]$ is the structural stiffness matrix, $[\mathrm{M}_s]$ is the structural mass matrix, $\{\mathrm{F}_s\}$ is a vector of applied structural loads, $\{U\}$ is a vector of unknown nodal displacements and hence $\{\ddot{U}\}$ is a vector of the second derivative of displacements with respect to time, equivalent to the acceleration of the nodes.

For pressure-formulated acoustic elements, the lossless finite element equation for the fluid in matrix form is

$$[\mathrm{M}_f]\{\ddot{p}\} + [\mathrm{K}_f]\{p\} = \{\mathrm{F}_f\} \quad (19)$$

where $[\mathrm{K}_f]$ is the equivalent fluid stiffness matrix, $[\mathrm{M}_f]$ is the equivalent fluid mass matrix, $\{\mathrm{F}_f\}$ is a vector of applied fluid loads, $\{p\}$ is a vector of unknown nodal acoustic pressures, and $\{\ddot{p}\}$ is a vector of the second derivative of acoustic pressure with respect to time.

The interaction of the fluid and structure occurs at the interface between the structure and the acoustic elements, where the acoustic pressure exerts a force on the structure and the motion of the structure produces a pressure. To account for the coupling between the structure and the acoustic fluid, additional terms are added to the equations of motion for the structure and fluid (of density ρ_0), respectively, as

$$[\mathrm{M}_s]\{\ddot{U}\} + [\mathrm{K}_s]\{U\} = \{\mathrm{F}_s\} + [\mathrm{R}]\{p\} \quad (20)$$

$$[\mathrm{M}_f]\{\ddot{p}\} + [\mathrm{K}_f]\{p\} = \{\mathrm{F}_f\} - \rho_0 [\mathrm{R}]^T \{\ddot{U}\} \quad (21)$$

where [R] is the coupling matrix that accounts for the effective surface area associated with each node on the fluid-structure interface. Equations (20) and (21) can be formed into a matrix equation including the efects of damping as

$$\begin{bmatrix} M_s & 0 \\ \rho_0 R^T & M_f \end{bmatrix} \begin{Bmatrix} \ddot{U} \\ \ddot{p} \end{Bmatrix} + \begin{bmatrix} K_s & -R \\ 0 & K_f \end{bmatrix} \begin{Bmatrix} U \\ P \end{Bmatrix} = \begin{Bmatrix} \mathrm{F}_s \\ \mathrm{F}_f \end{Bmatrix} \quad (22)$$

For harmonic analyses, this equation can be reduced to an expression without differentials as

$$\left(-w^2 \begin{bmatrix} M_s & 0 \\ \rho_0 R^T & M_f \end{bmatrix} + \begin{bmatrix} K_s & -R \\ 0 & K_f \end{bmatrix} \right) \begin{Bmatrix} U \\ P \end{Bmatrix} = \begin{Bmatrix} \mathrm{F}_s \\ \mathrm{F}_f \end{Bmatrix} \quad (23)$$

The important feature to notice about Equation (23) is that the matrix on the left-hand side is unsymmetric and solving for the nodal pressures and displacements requires the inversion of this unsymmetric matrix, which requires a signifcant amount of computer resources.

D. Clarinet tube model

For the modal and harmonic analyses in ANSYS the clarinet tube part is modeled with the parameters in table 2.

TABLE II. TUBE MODEL GEOMETRICAL AND ACOUSTIC PARAMETERS

Description	Parameter	Value	Units
Diameter	$2a$	15	mm
Length	L	600	mm
Speed of sound	c_0	343	m/s
Density	ρ_0	1.21	kg/m^3

E. Reed Model

First of all, the existing geometrical parameters [6, p.26] were used. After considering different alternatives, a model consisted of equations [Annex A] was taken into account, identifying the parameters with the limits and the coefficients of the functions. These functions were included in Matlab and the Fig. 56 was obtained.

The main problem of this option lies in the difficulty of including a geometry defined by points in a solid body, which allows the calculation of the natural frequencies and the vibration modes.

Therefore, other options were considered. To facilitate the modelling the "Blue Box" reed by Vandoren Paris was taken as a reference, and the basic parameters where extracted.

TABLE III. REED GEOMETRICAL PARAMETERS

Parameter	Measured value (mm)	Value range (mm)
L1 Stock length	35	[11-35]
L2 Vamp length	32	[20-60]
R Tip radius	7	[7-14]
B1 Width at heel	6	[3-11]
B2 Width at tip	6.5	[3.5-11.5]
T1 Heel thickness	2.7	[1-4.5]
T2 Tip thickness	0.1	[0.1-3.6]

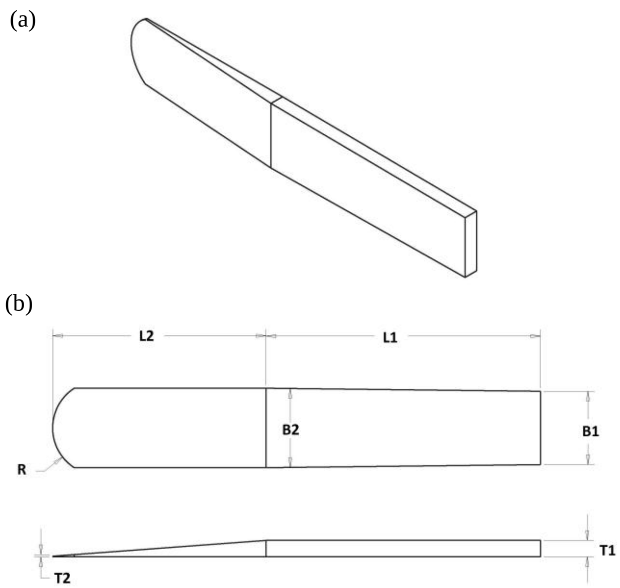


Fig. 3. Reed 3D representation (a) and its geometrical parameters (b).

In the last column of the table 3 the range of each parameter has been included. This information will be useful for the sensitivity analysis that will be introduced afterwards.

F. Reed Material

The physical properties of the reed are summarized below. The reed is made of the material named Arundo Donax [6, p.15], which is considered as an orthotropic elastic material.

TABLE IV. REED MATERIAL PROPERTIES

Elastic Modulus	Poisson Coefficient	Shear Modulus
$E_x = 14000\ MPa$	$v_{xy} = 0.22$	$G_{xy} = 1100\ MPa$
$E_y = 480\ MPa$	$v_{xz} = 0.22$	$G_{xz} = 1200\ MPa$
$E_z = 480\ MPa$	$v_{yz} = 0.22$	$G_{yz} = 1200\ MPa$
Density: $\rho = 520\ kg/m^3$		

G. Experimental Data

The experimental data acquisition has been done by using the "Conn-Selmer Prelude Stuent Model CL711 Bb Clarinet" as well as MATLAB.

The elaborated script in MATLAB allows to perform 5 seconds recording by making use of the computer's microphone. This audio file is filtered by the "Direct Form II Transposed" method to avoid possible errors caused by noise. After the filtering, the fast fourier transform (FFT) is made and the natural frequencies of the signal are identified.

Given that the most relevant frequency is the one corresponding to the first vibration mode, the value of the frequency must be perfectly characterized. Therefore, the maximum value of the frequency response is identified and a first order gaussian fitting is done in a 250 Hz range. Consequently, a gaussian curve which average value corresponds to the first natural frequency is obtained.

In order to validate this experimental data, a musical database is considered. This musical database registers all possible frequencies of every note. It is important to explain that each note corresponds to an only air vibration frequency. However, the performed note in the Bb clarinet does not correspond with the sounded one. The reason is that the Bb clarinet is a transposing instrument. When the note C is played in a Bb clarinet, the instrument will actually sound a Bb.

By making this slight modification, the obtained natural frequencies are compared with the database frequencies. The note with the closest frequency is obtained, therefore, it is verified that the experimental data have been correctly filtered and analyzed. The errors between the real frequency and the database frequency are small, but this is work does not intend to explain these errors.

These experimental data will subsequently serve to compare the clarinet real waveform with the one obtained in ANSYS.

III. RESULTS

A. Structural modal analysis of the reed

The clarinet reed model was based in the "Blue Box" reed by Vandoren Paris. In this analysis a good mesh was essential for the goodness of the results (Fig.4). The natural frequencies (Fig. 5-9) and the frequency response with a normal force applied in the extreme of the reed (Fig. 10) were calculated using ANSYS.

Later this FRF will be compared with the one obtained in the complete model. Moreover, the deformation for the first 5 modes has been represented in Fig. 5-9.

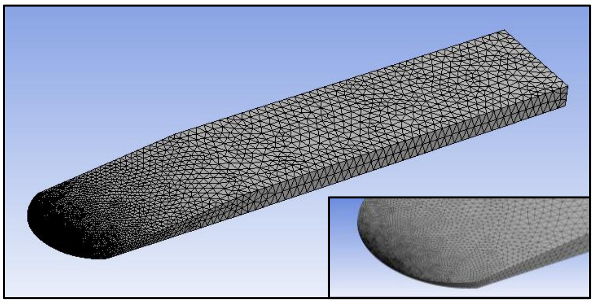


Fig. 4. Clarinet reed mesh: curvature and proximity advanced size functions

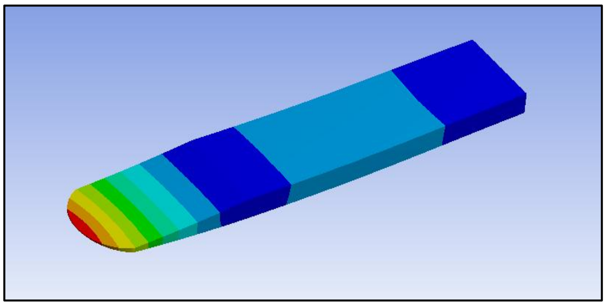


Fig. 5. First mode of vibration of the reed

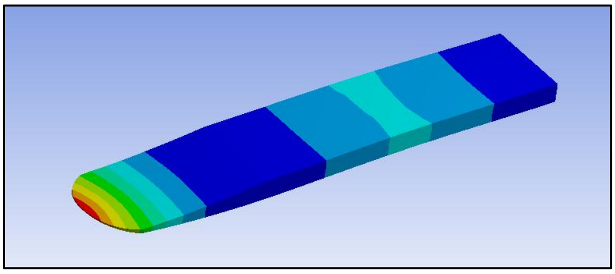


Fig. 6. Second mode of vibration of the reed

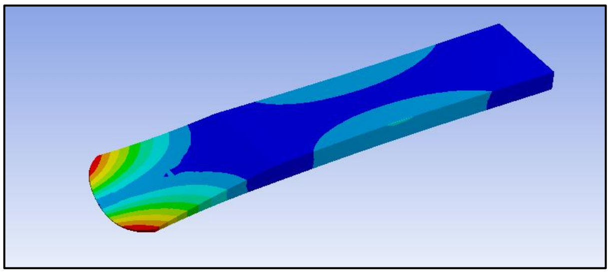


Fig. 7. Third mode of vibration of the reed

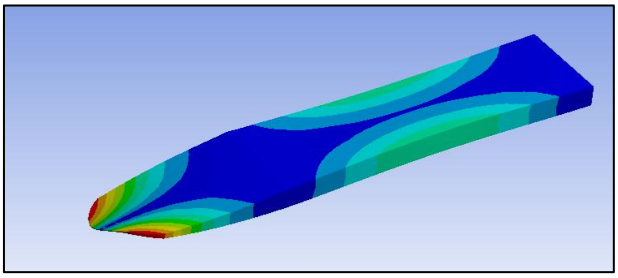


Fig. 8. Fourth mode of vibration of the reed

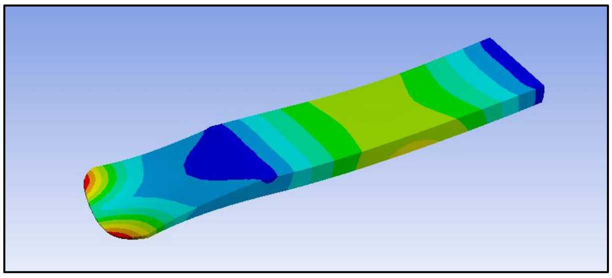


Fig. 9. Fifth mode of vibration of the reed

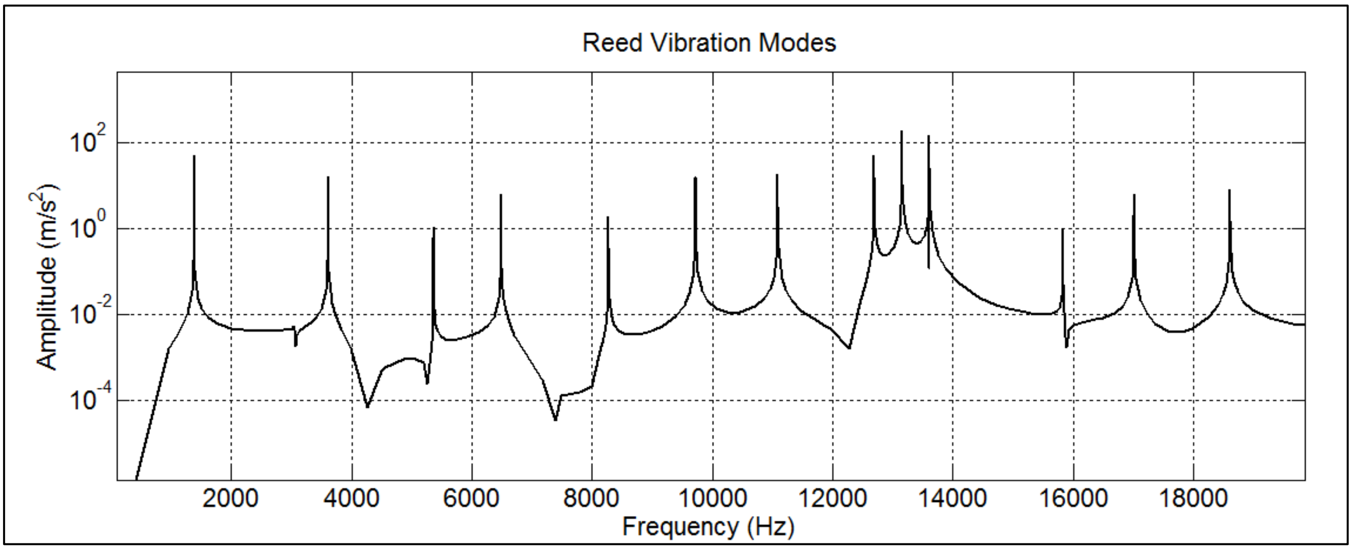
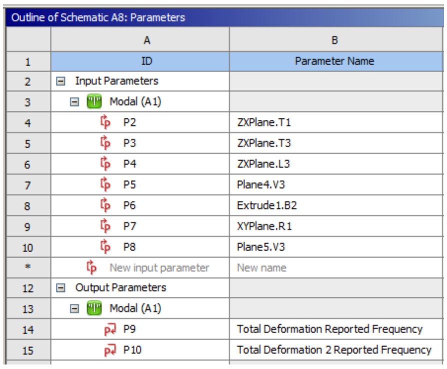


Fig. 10. Harmonic response of the reed with a normal force applied in the extreme of the reed.

B. *Sensitivity analysis of the parameters of the reed in its natural frequencies*

By making a parametric model of the reed, a sensitivity analysis can be carried out (Table 5) by creating some design points in ANSYS (Fig. 11). The x axis accounts for the modified dimension, and the y axis for the percentage change of the natural frequency with respect to the original natural frequency (Fig. 12-21). The change in the first mode and in the second mode have been calculated, due to their relevance in the clarinet acoustics.

A change in the first natural frequency affects the behavior of the clarinet, since it is a critical frequency for the reed and of course for the quality of the sound of the clarinet.


Outline of Schematic A8: Parameters

	A	B
1	ID	Parameter Name
2	⊟ Input Parameters	
3	⊟ Modal (A1)	
4	P2	ZXPlane.T1
5	P3	ZXPlane.T3
6	P4	ZXPlane.L3
7	P5	Plane4.V3
8	P6	Extrude1.B2
9	P7	XYPlane.R1
10	P8	Plane5.V3
*	New input parameter	New name
12	⊟ Output Parameters	
13	⊟ Modal (A1)	
14	P9	Total Deformation Reported Frequency
15	P10	Total Deformation 2 Reported Frequency

Fig. 11. Parametric model of the reed in ANSYS

TABLE V. PARAMETERS AND NATURAL FREQUENCIES CORRELATIONS

Parameters	Correlation
L1	Critical dimension. High negative correlation with both modes (60 % change in frequencies).
L2	As L2 increases, the natural frequencies decrease. Same as L1.
R	An increase from 8 to 10 mm produces a sudden change in the frequencies (5%).
B1	The initial B1 data represents a maximum in the frequency range. Good design.
B2	Behavior not clear in the first mode. The maximum of the second mode appears in 1.03 times the original frequency.
B1 and B2	Changing both widths simultaneously produces a positive correlation in the frequencies.
T1	Increasing the thickness of the heel affects highly the frequencies, with a high positive correlation (105 % change in frequencies).
T2	Opposite behavior in T2. The correlation is negative for the first mode, but not so high, 40% changes. The second mode remains mostly constant.
T2 in detail	Little variations produce a decrease in the frequencies, but only of 8 %.
T1 and T2	Increasing the overall thickness increases considerably the frequencies. Same effect as changing only T1.

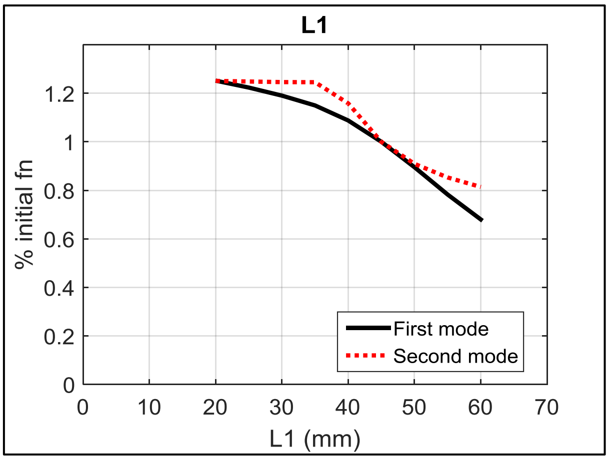


Fig. 12. Reed natural frequencies and L1 [11-35] mm parameter correlation

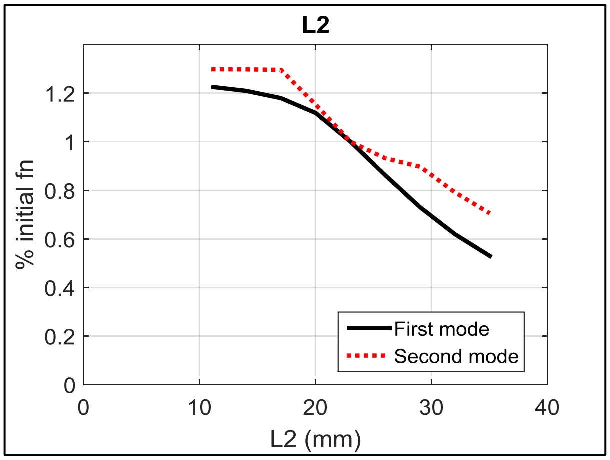


Fig. 13. Reed natural frequencies and L2 [20-60] mm parameter correlation

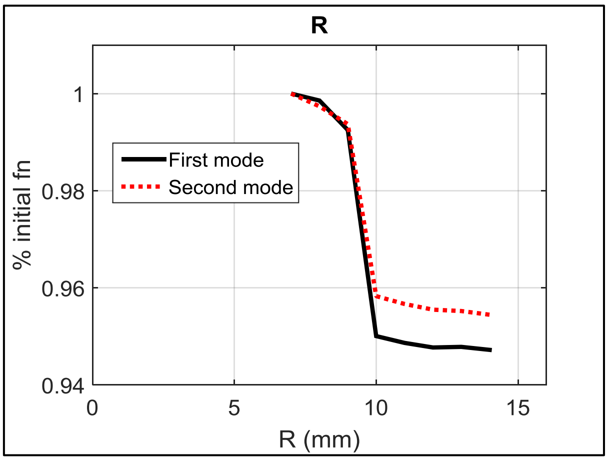


Fig. 14. Reed natural frequencies and R [7-14] mm parameter correlation

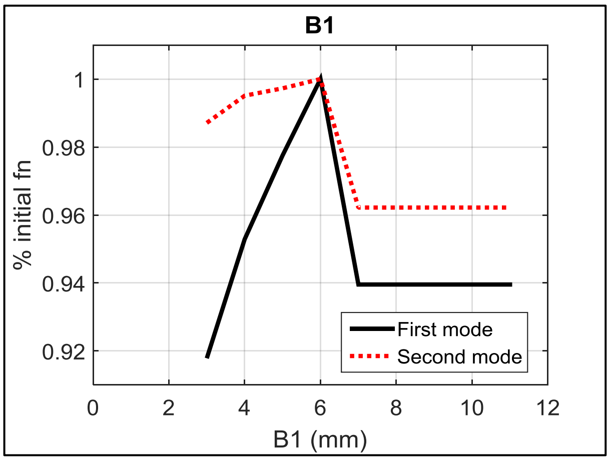


Fig. 15. Reed natural frequencies and B1 [3-11] mm parameter correlation

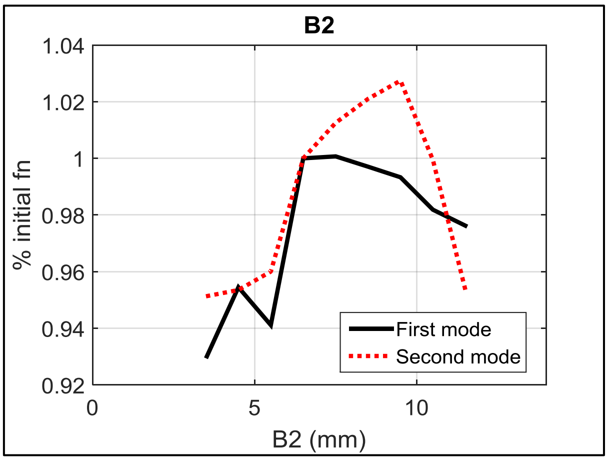


Fig. 16. Reed natural frequencies and B2 [3.5-11.5] mm parameter correlation

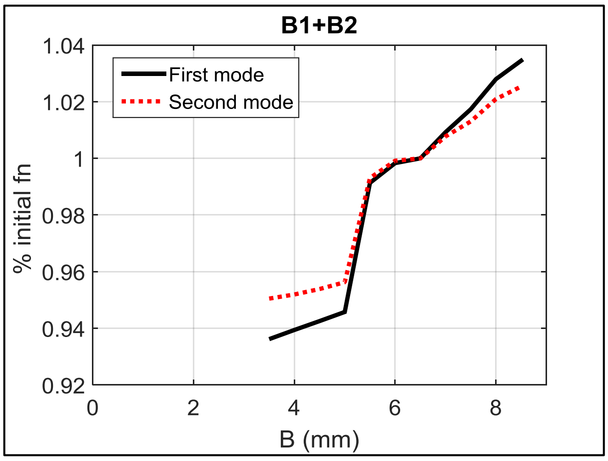


Fig. 17. Reed natural frequencies and B1 and B2 parameters correlation

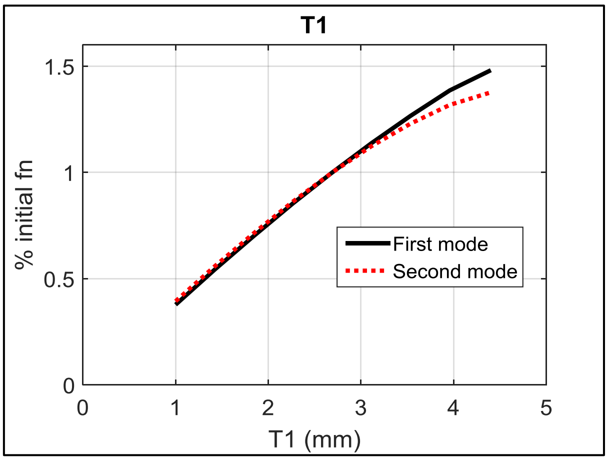


Fig. 18. Reed natural frequencies and T1 [1-4.5] mm parameter correlation

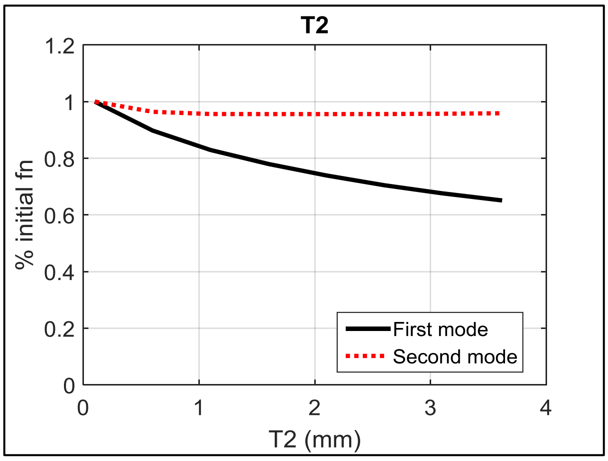


Fig. 19. Reed natural frequencies and T2 [0.1-3.6] mm parameter correlation

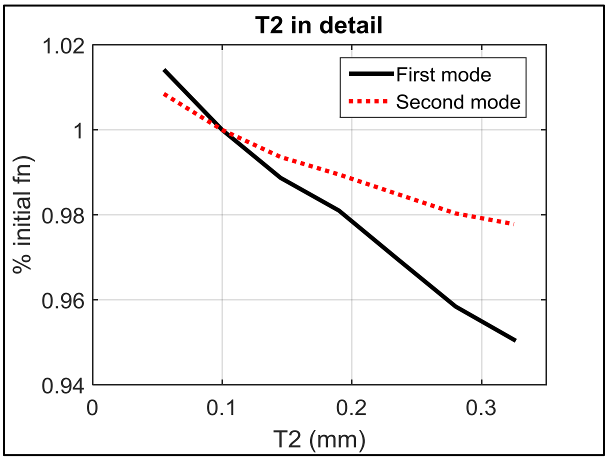


Fig. 20. Reed natural frequencies and T2 [0.01-0.325]mm parameter correlation

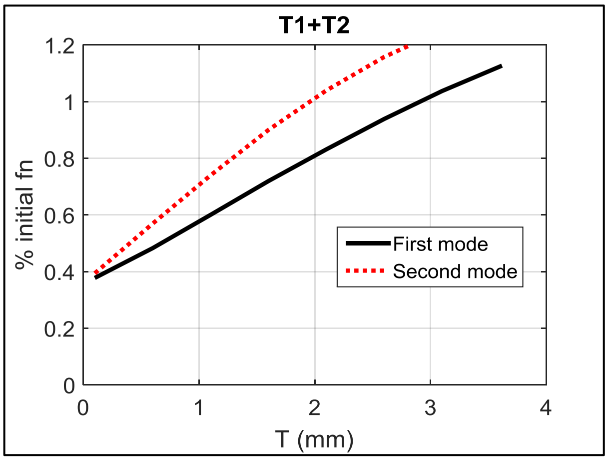


Fig. 21. Reed natural frequencies and T1 and T2 parameters correlation

C. Acoustic modal analysis of the clarinet tube

The clarinet tube model is a 0.53 long tube. This model is very similar to the complete model, and the results are almost identical. Therefore, for this paper only the complete clarinet model results will be displayed.

For future works, it could be appropriate to study the tube acoustics for more than one musical note. For that reason, the clarinet hole´s position was measured in the real clarinet, and introduced in the ANSYS model (Fig. 22). Applying a 0 Pa acoustic pressure in one of those holes will reduce the wavelength, and thus increase the frequency, creating a different musical note.

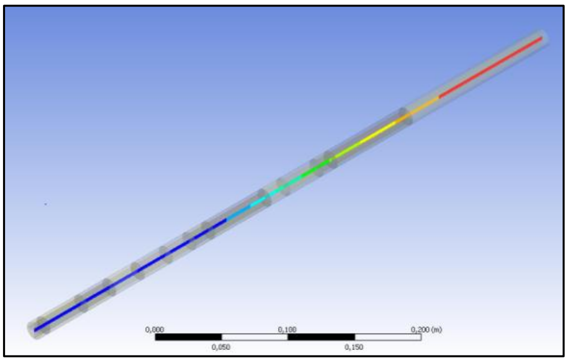


Fig. 22. Clarinet tube model, with the holes position along the tube

D. Experimental data

The audio of the musical note E3 has been filtered and characterized with a MATLAB script. In Fig. 23 the original and the filtered signal have been illustrated, and in Fig. 24 the same data zoomed. The filter has suppressed interfering signals and has reduced the background noise.

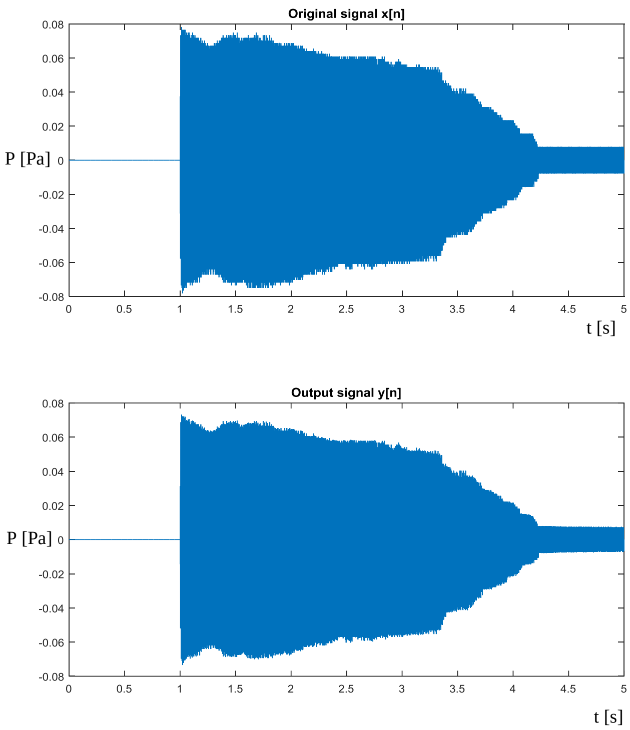


Fig. 23. Clarinet E3 musical note 5 second recording: original and filtered.

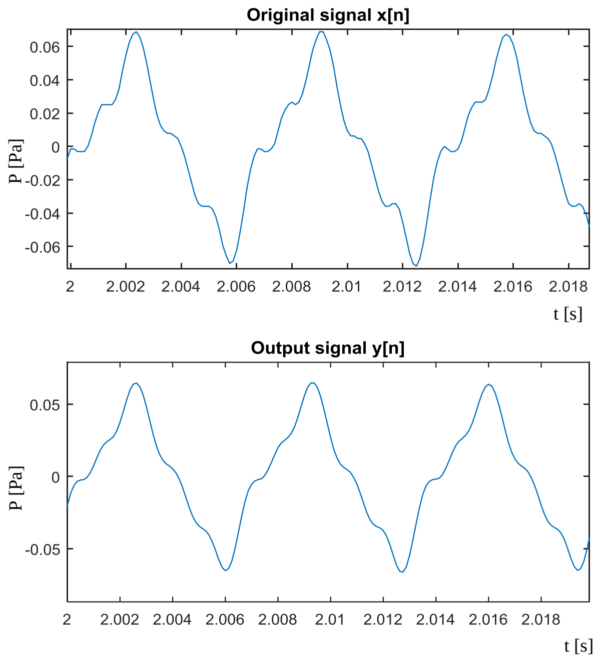


Fig. 24. Clarinet E3 musical note 5 second recording: original and filtered.

The FFT amplitude and phase were also calculated, represented in Fig. 25. Then, the modes were identified and fitted (Fig. 26). In order to validate the obtained natural frequency, a spectrogram of the frequency versus the time was represented (Fig. 27), checking of the stability of the modes.

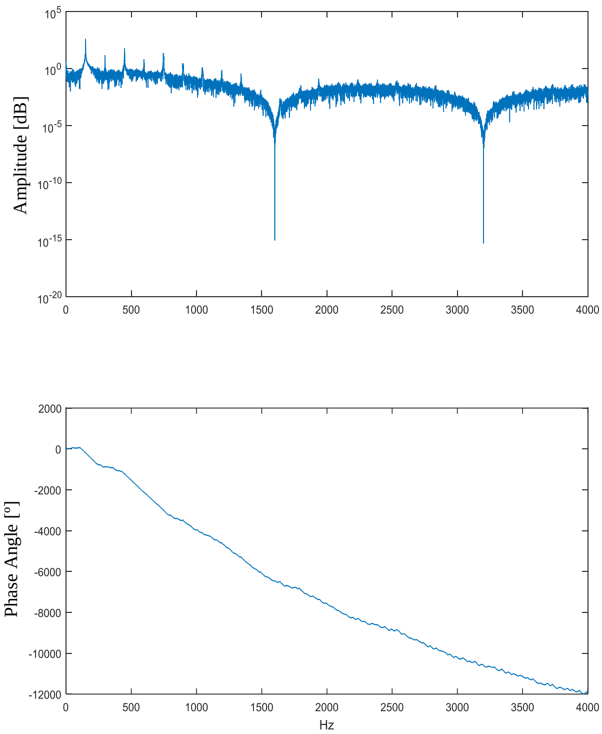


Fig. 25. E3 sound signal FFT amplitude and phase angle

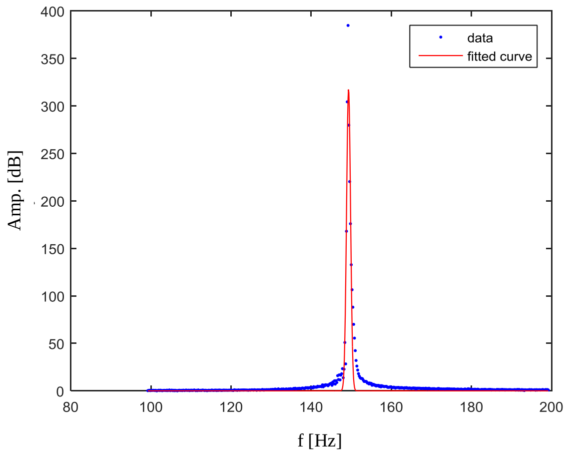


Fig. 26. First order gaussian fitting of the first natural frequency.

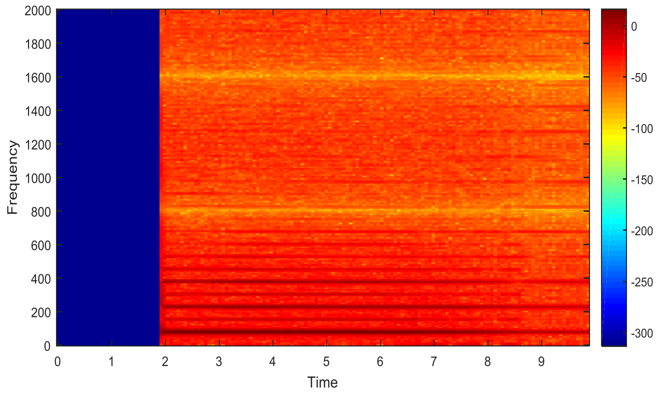


Fig. 27. Spectrogram of the frequency versus the time of the filtered signal

E. *Theoretical pressure and velocity distributions.*

Using the Four-Pole Method, the pressure distribution along the clarinet cylindrical duct is work out by a script in MATLAB. A driving volume velocity is applied at one end and the other end is closed. This results have been obtained using the frequencies of the vibration modes of the simulation in ANSYS. It is strongly recommended to compare the pressure distributions for the 1st,3rd and 5th with the Fig. 1 included in the "Methodology" section.

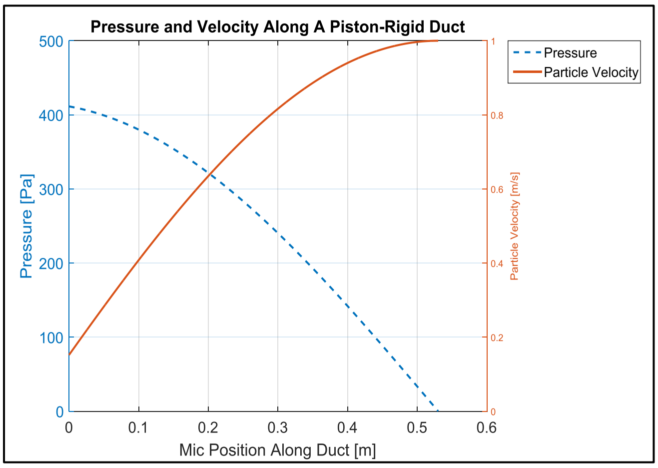


Fig. 28. Pressure and velocity distribution in the tube for the 1st harmonic

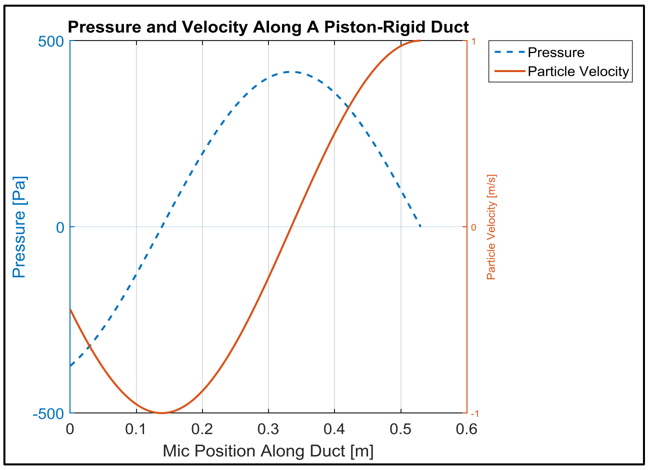


Fig. 29. Pressure and velocity distribution in the tube for the 3rd harmonic

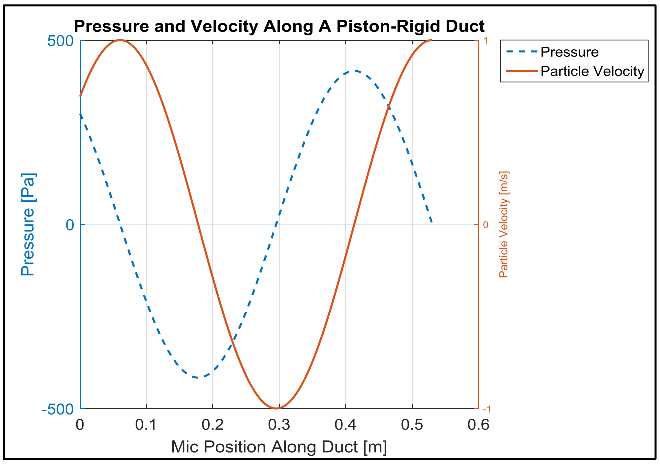


Fig. 30. Pressure and velocity distribution in the tube for the 5th harmonic

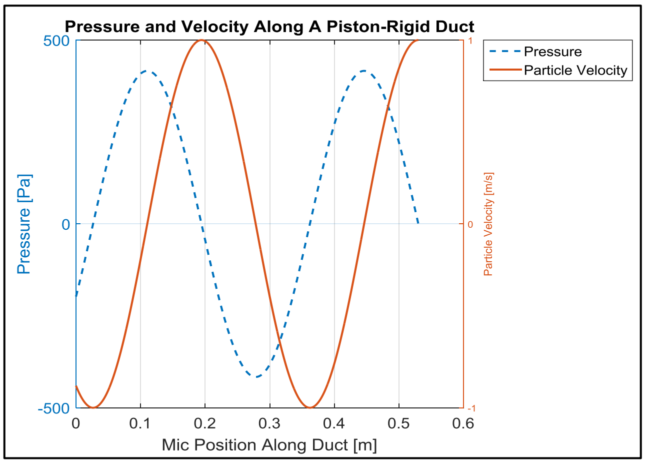


Fig. 31. Pressure and velocity distribution in the tube for the 7th harmonic

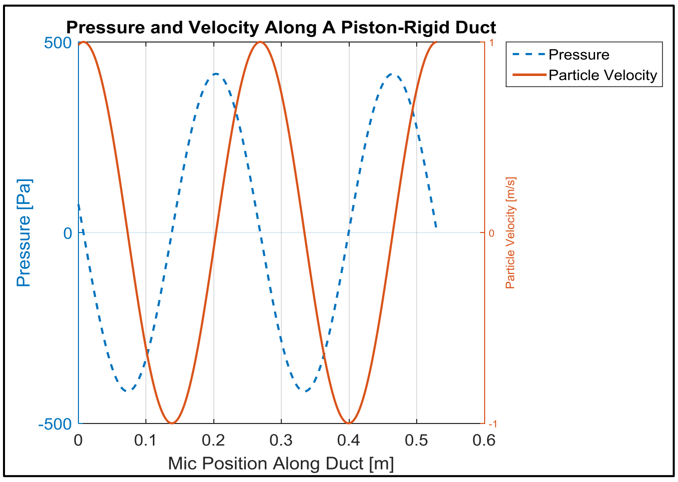


Fig. 32. Pressure and velocity distribution in the tube for the 9th harmonic

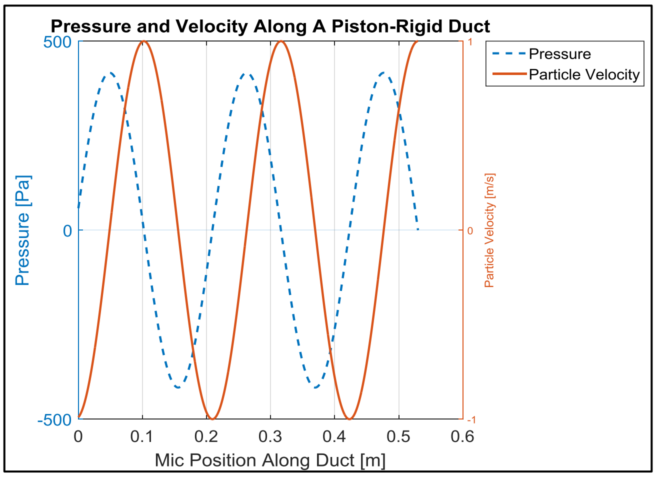


Fig. 33. Pressure and velocity distribution in the tube for the 11th harmonic

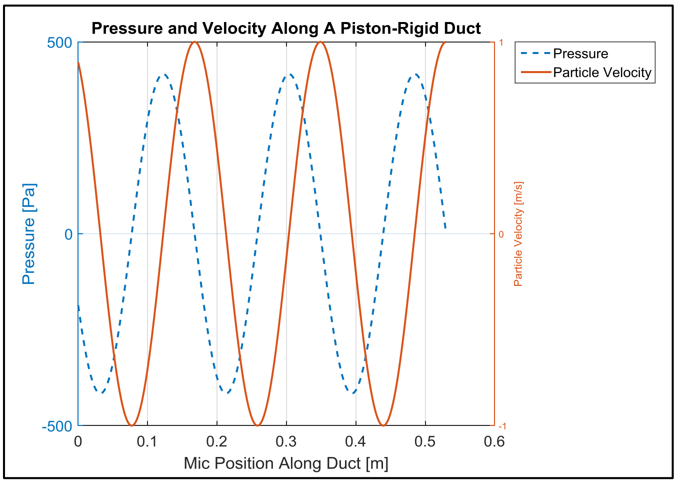


Fig. 34. Pressure and velocity distribution in the tube for the 13th harmonic

F. *Acoustic and structural modal analysis of the clarinet: tube + reed*

The clarinet ANSYS model is a 0.53 meters long tube with the designed mouthpiece and the reed coupled in one end. The sweep method is used for meshing the tube, and a small element size is applied in the reed (Fig. 35). This model accounts for the fluid-structure interaction between the reed and the air in the rest of the tube. The air in the tube is considered as an acoustic body, and an acoustic pressure of 0 Pa is applied in the free end. Then, a modal simulation and the harmonic response of the reed are carried out.

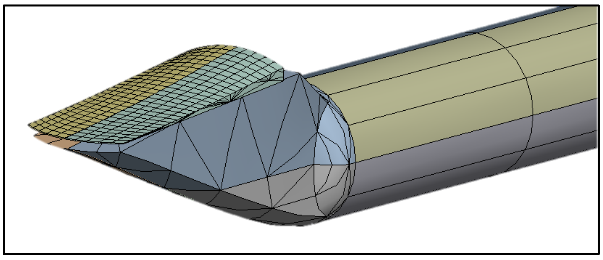


Fig. 35. Clarinet tube and reed mesh in ANSYS

With this, the pressure distributions for each mode can be extracted from ANSYS. The pressure distributions shown in Fig. 36 correspond to the average pressure in each point of the axis of the tube. The amplitudes of these modes are not representative of the real model, since the simulations have been performed without damping, given that the important information are the natural frequencies.

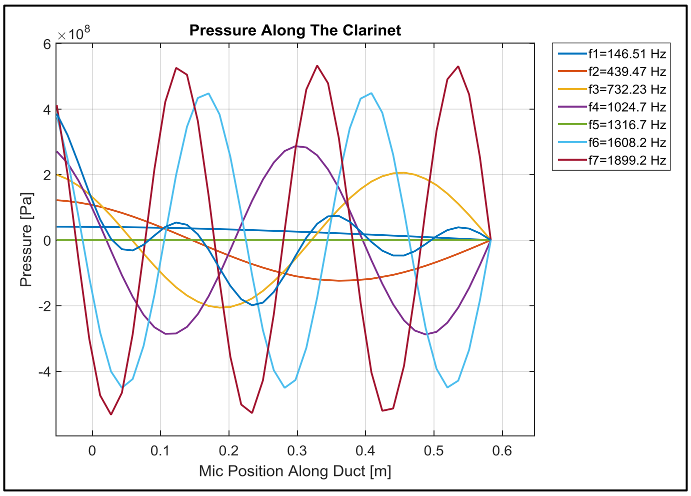


Fig. 36. Pressure distributions for the 7 first modes of the air column

However, the pressure distributions are useful if are normalized. Therefore, after normalizing each mode by their maximum value, the Fig. 37 distributions are obtained.

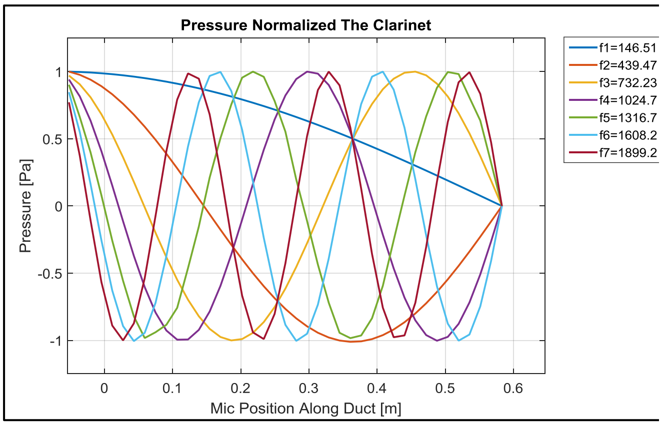


Fig. 37. Normalized pressure distributions for the 7 first modes of the air column

Figures 38-46 include the acoustic pressure for the first 9 modes found in ANSYS. Within each figure, the corresponding mode has been indicated. It must be pointed out the special case of the mode number 6. As can be seen in Fig. 43, the acoustic pressure does not follow any logical distribution, since this frequency corresponds with one of the natural frequencies of the reed. The total deformation of the reed in this frequency is illustrated in Fig 47.

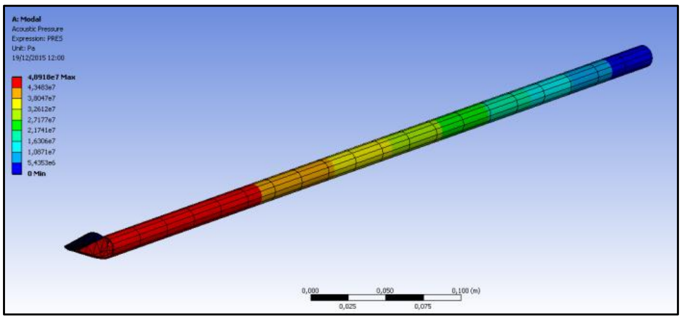


Fig. 38. Acoustic pressure for the mode Nº 1: (1st mode of the air column)

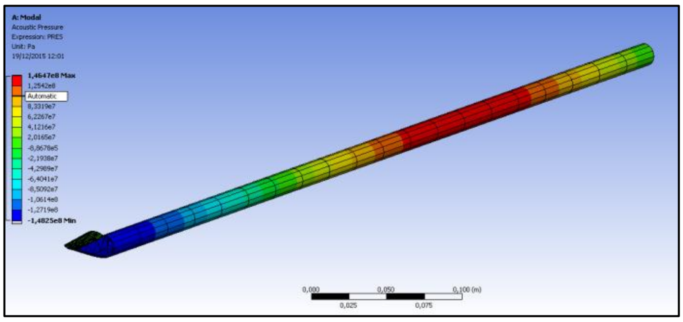


Fig. 39. Acoustic pressure for the mode Nº 2: (3st mode of the air column)

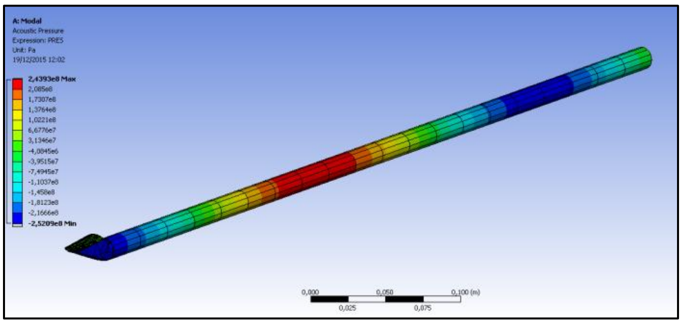


Fig. 40. Acoustic pressure for the mode Nº 3: (5st mode of the air column)

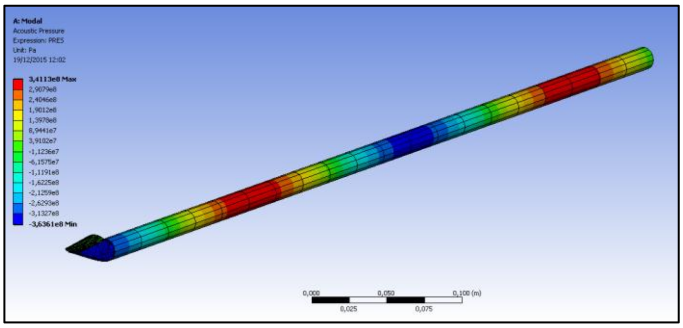


Fig. 41. Acoustic pressure for the mode Nº 4: (7st mode of the air column)

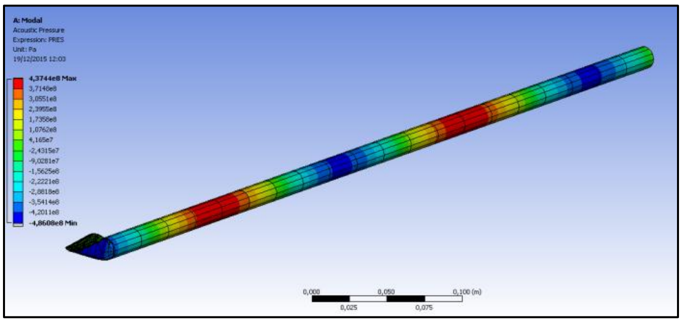


Fig. 42. Acoustic pressure for the mode Nº 5: (9st mode of the air column)

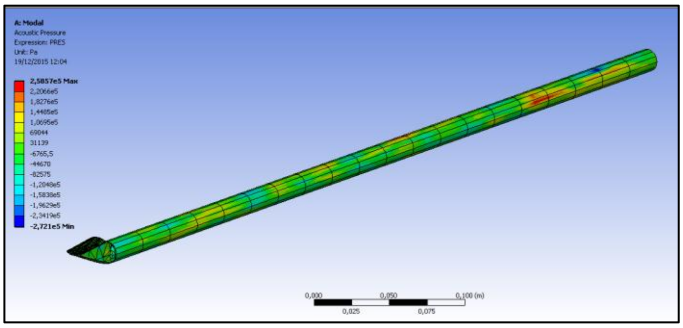


Fig. 43. Acoustic pressure for the mode Nº 6: (1st mode of the reed)

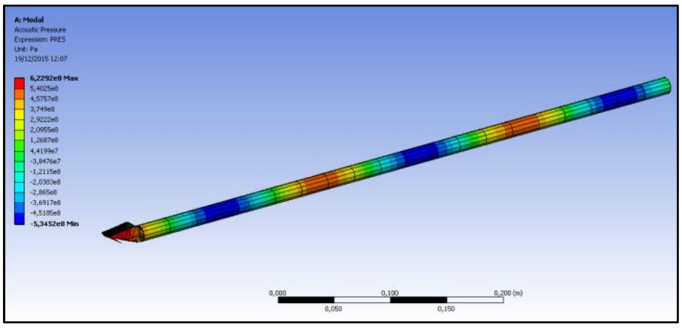


Fig. 44. Acoustic pressure for the mode Nº 7: (11st mode of the air column)

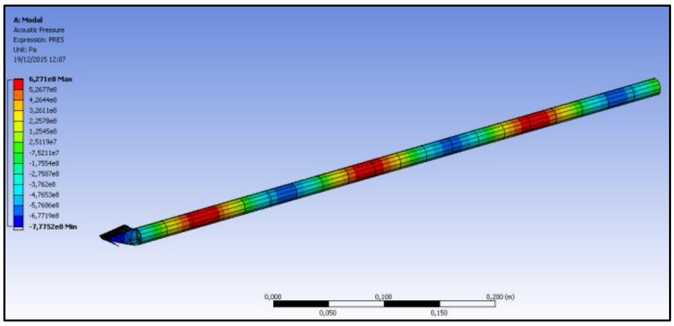


Fig. 45. Acoustic pressure for the mode Nº 8: (13st mode of the air column)

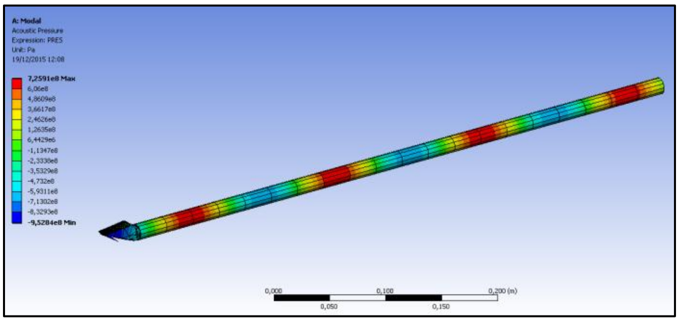


Fig. 46. Acoustic pressure for the mode Nº 9: (15st mode of the air column)

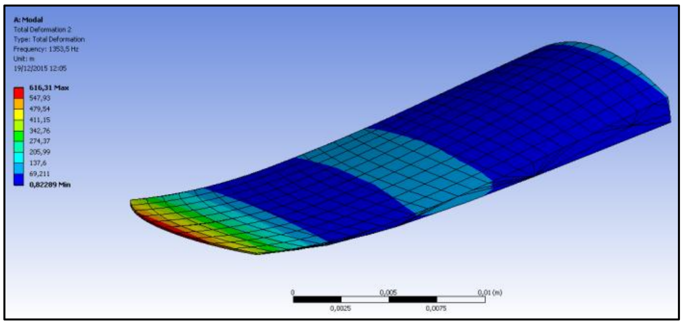


Fig. 47. Total deformation of the reed in mode Nº 6 : (1st mode of the reed)

After the modal analysis, the harmonic response of the reed was calculated. In this modal analysis only 20 modes were explored, the majority of them related to the air column. That is why the harmonic response of the reed with the tube only shows 2 modes. In Fig. 48, a comparison between the two cases is represented. It can be concluded that the first and the second modes are almost coincident in both cases. Therefore, with this result, a change in the natural frequencies of the reed will also change the amplitude of the pressure applied in the mouthpiece and therefore in the note played.

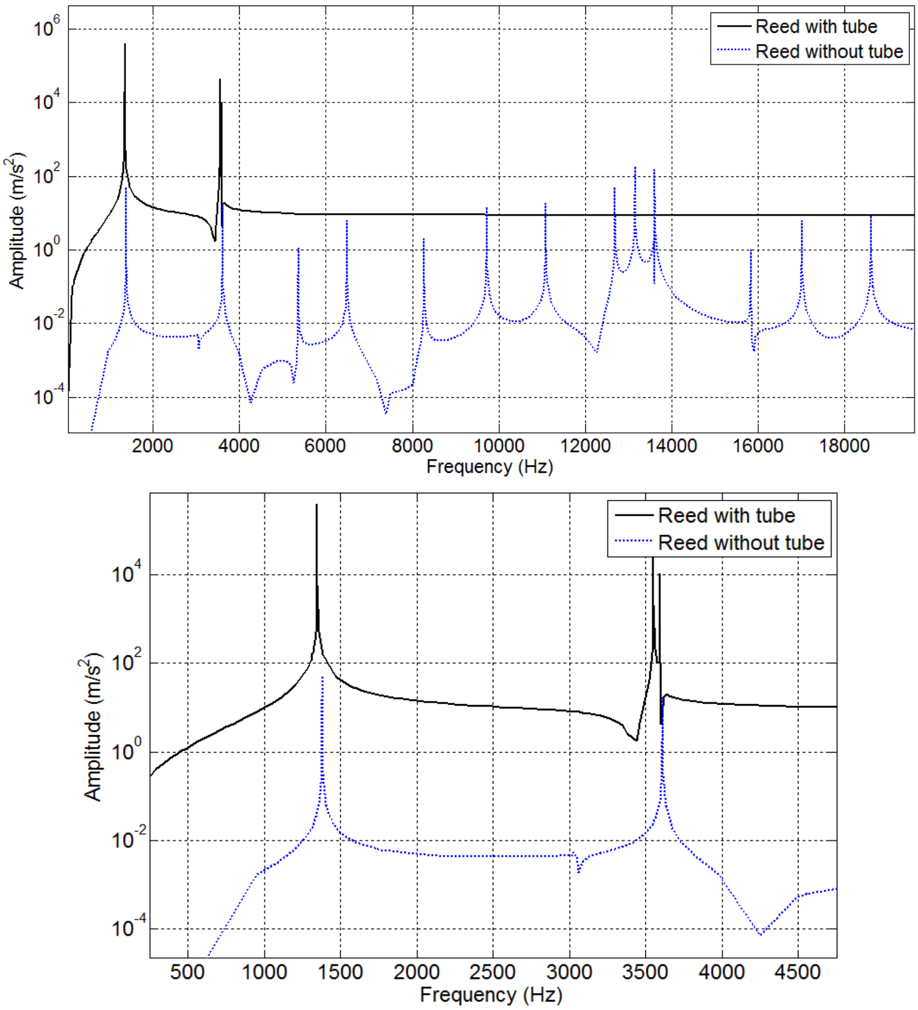


Fig. 48. Harmonic response of the reed with the tube and without the tube.

G. Clarinet waveform study

From the experimental data (Fig. 25) the amplitude p_i of each harmonic can be extracted. The amplitude represents the coefficient of the Fourier series, so that the sum of all the harmonics makes the final signal and therefore the waveform of the clarinet sound.

$$p(t) = p_1 \sin(wt) + p_3 \sin(3wt) + \cdots$$
$$\cdots + p_5 \sin(6wt) + p_7 \sin(7wt) + \cdots \quad (24)$$

With this, only a quarter of the wave is calculated, and the rest is extrapolated, following a sinusoidal form (Fig. 49).

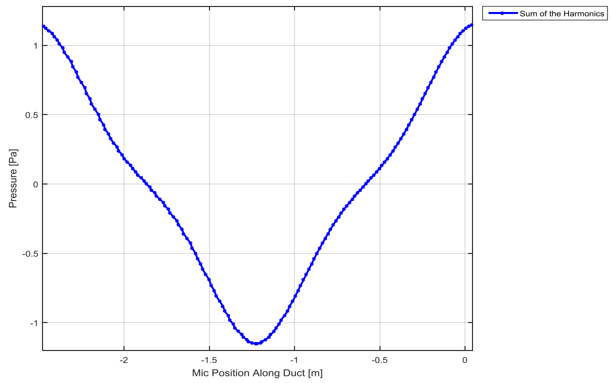


Fig. 49. Total sound pressure level along the tube

In Fig. 24 the waveform of the clarinet was represented. By using the data from Fig. 37 and the amplitudes of the Fourier series, an estimation of the waveform was developed. In figures 50 and 51 the comparison between the experimental data wave and the ANSYS-Fourier wave is represented, where the difference error is not notably high.

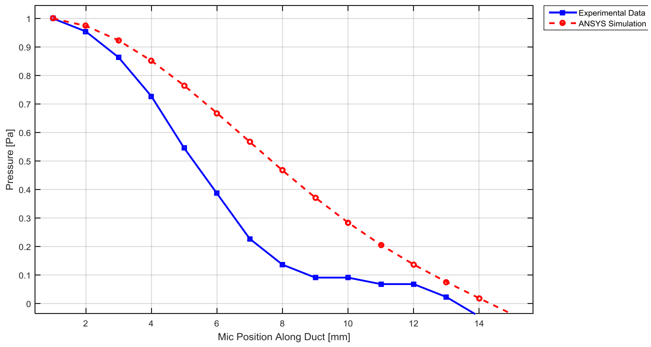


Fig. 50. Pressure level comparison in the tube: Experiment vs simulation

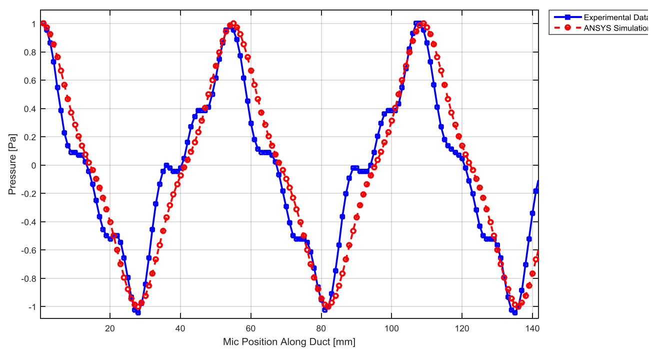


Fig. 51. Extrapolated pressure level comparison in the tube: Experimental data vs ANSYS simulation

H. Inharmonicity study

Musical instruments such as the clarinet do not exhibit ideal harmonics because they have a more complex geometry than a cylindrical pipe. Inharmonicity is a measure of the deviation of each harmonic from its theoretical frequency. The percent deviation $\Delta_n$is calculated as follows:

$$\Delta_n = 100 \left(\frac{n_{exp} - n_{thy}}{n_{thy}} \right) \quad (25)$$

where n_{exp} is the experimental ratio between two consecutive harmonic frequencies and n_{thy} is the theoretical ratio between the same consecutive harmonic frequencies.

The tables 6 and 7 summarize the natural frequencies obtained in the ANSYS simulation and in the experimental data. The simulation carried out does no account the even modes, only the odd ones, since it is an ideal case. This fact was introduced in the first section.

TABLE VI. NATURAL FREQUENCIES AND HARMONICITY DEVIATION IN ANSYS SIMULATION

	ANSYS Freq.	n_{thy}	n_{exp}	Δ_n
1	146.51	1		
3	439.47	3.000	3.000	-0.01%
5	732.23	1.667	1.666	-0.03%
7	1024.7	1.400	1.399	-0.04%
9	1316.7	1.286	1.285	-0.06%
11	1608.2	1.222	1.221	-0.07%
13	1899.2	1.182	1.181	-0.07%

TABLE VII. NATURAL FREQUENCIES AND HARMONICITY DEVIATION IN EXPERIMENTAL DATA

	Experimental Freq.	n_{thy}	n_{exp}	Δ_n
1	149.2			
2	298.2	2.000	1.999	-0.07%
3	447	1.500	1.499	-0.07%
4	596.5	1.333	1.334	0.08%
5	745.5	1.250	1.250	-0.02%
6	894	1.200	1.199	-0.07%
7	1045	1.167	1.169	0.19%
8	1194	1.143	1.143	-0.02%

With this data, an estimation of the inharmonicity of both cases was calculated and represented in figures 52 and 53. It must be pointed out that simulation inharmonicity increases as the vibration mode increases, while with experimental data there is not a clear trend. Even so, the obtained errors are nearly negligible, below 0.2%.

A visual comparison of the odd modes frequencies is represented in figure 54, while in figure 55 the percentage of error is visualized. In this term, the maximum error is less than 2%, thus both data have been certainly well obtained and analyzed.

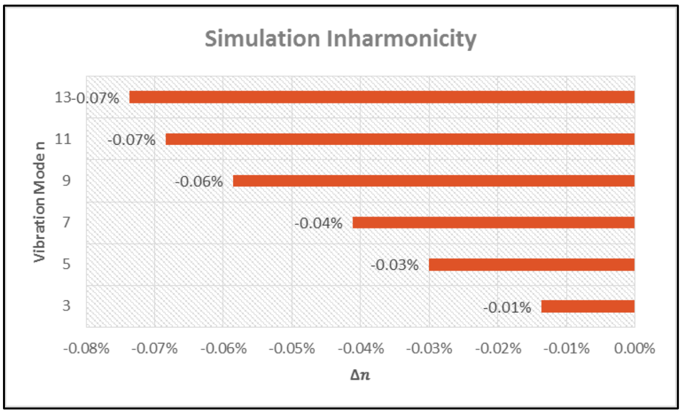


Fig. 52. ANSYS simulation inharmonicity deviation in each found mode

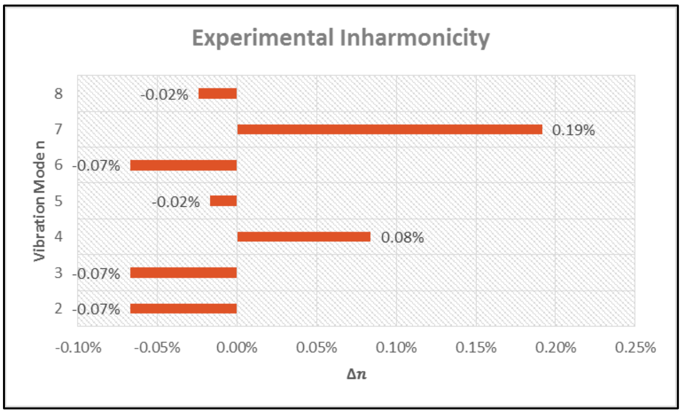


Fig. 53. Experiment inharmonicity deviation in each found mode

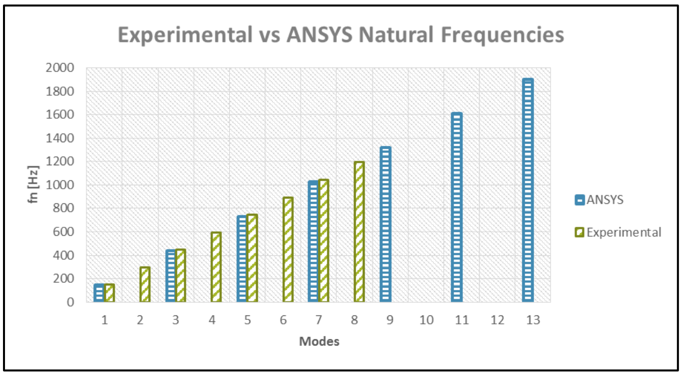


Fig. 54. Natural frequencies comparison between experimental and simulation

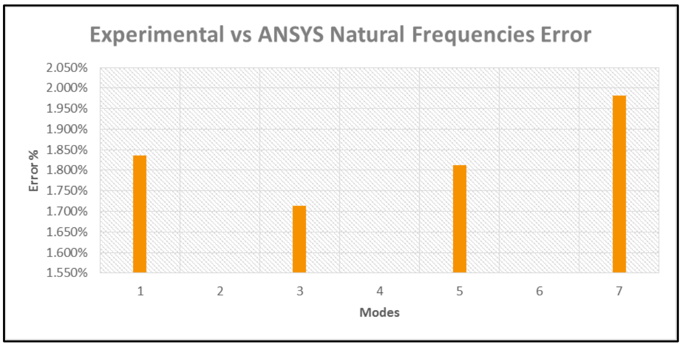


Fig. 55. Natural frequencies percentage error between experimental and simulation

IV. CONCLUSIONS

With this research an understanding of the mode of operation of the clarinet has been gained. Overall, this work has studied the reed from the point of view of vibrations.

Paying attention to the objectives introduced at the beginning of this article, it can be pointed out that:

- ✓ An acoustic model of a clarinet has been carried out, and the model is valid to do a modal analysis, as much from the reed as from the air column in the clarinet.
- ✓ An modal analysis of the reed has succesfully been developed and its harmonic response has been obtained. The model has been used for performing the expected parametric analysis, from which some conclusions can be extracted:
  - o First, it must be pointed out that this study has been done following one set of parameters, and that with other set of them, the results could vary slightly.
  - o Even so, it is important to stand out the critical parameters for the natural frequency of the reed. The dimensions L1, L2, T1 and T2 modify excessively the value of the natural frequency, in a average range of 50%. This fact could provoke a highly reduced natural frequency that could intersect with the frequency range of musical notes. Consequently, the reed could enter in resonance if a note frequency is the same as the natural of the reed.
  - o That is why it has been pointed out previously that the parameter B1 has a good design valur, since any other value reduces the natural frequencies and complicates the normal behaviour of the reed.
- ✓ Regarding the experimental data, these have been filtered correctly and the FFT has been properly carried out. That is why comparing the theoretical results with the experimental ones (Fig.54), the errors are so small (Fig.55).
- ✓ However, it must be pointed out thar the theoretical model is not able to identify the even vibration modes of the air column, due to the fact that ideally these modes do not exist, like it has been explained in the section 2.A.
- ✓ On the other hand, by means of the theoretical modal analysis, the Four-Pole method has been used for calculating the pressure and velocity distributions with the data from ANSYS. The results have been satisfactory, and the comparison between both theoretical models, the Four-Pole and the ANSYS modes is coherent.
- ✓ From the theoretical modes the aproximate waveform of the clarinet sound has been obtained. In the section The theoretical simualtion of the modes has been used, and by means of the amplitudes of the experimental modes, a very similar the waveform to the clarinet real one has been obtained.

- ✓ Finally, it should be noted that after performing the study of harmonicity, the experimental data are valid and agree with the theoretical ones.

With all said, this work has verified that the open-closed end tube model is apporpiate for simulating the behaviour of the clarinet, and that the natural frequencies of the reed are very dependent on its geometrical parameters.

For future work, it is important to point out that this work has not be able to relate satisfactorily the value of the natural frequency of the reed with the sound created in the clarinet.

It is true that a change in the natural frequency of the reed will vary the behaviour of the reed and the amplitude of its deformation. With that, it will also change the volume of air that will flow through the space between the mouthpiece and the reed and that the pressure in the mothupiece will change as well. It can be asserted that the volume of the produced sound depends on the oscillation frequency of the reed, but its influence in the quality of the sound is set aside for future work.

V. BIBLIOGRAPHY

[1] K. Holz, “The Acoustics of the Clarinet: An Observation of Harmonics, Frequencies, Phases, Complex Specific Acoustic Impedance, and Resonance”, Section I.

[2] Backus, "Small Vibration Theory of the Clarinet," J. Acoust. Soc. Am. 35, 305-313 (1963).

[3] S. E. Stewart, W. J. Strong, “Functional model of a simplified clarinet”, 1979.

[4] P. Picart, J. Leval, F. Piquet, J.P. Boileau, J.P. Dalmont, “Analysis of Clarinet Reed Oscillations With Digital Fresnel Holography”.

[5] V. Chatziioannou, M. van Walstijn, “Reed vibration modelling for woodwind instruments using a two-dimensional finite difference method approach”, 2007.

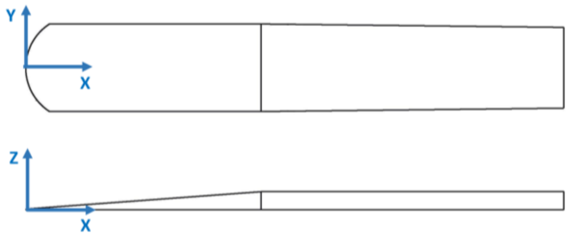
[6] P.A. Taillard, F. Laloë, M. Gross, J.P. Dalmont and J. Kergomard, “Statistical estimation of mechanical parameters of clarinet reeds using experimental and numerical approaches”, 2014, pp: 15,25,26

[7] S.W. Rienstra & A. Hirschberg, “An Introduction to Acoustics”, pp: 22.

[8] C. Q. Howard and B. S. Cazzolato, “Acoustic Analyses Using Matlab® and Ansys®”, CRC Press 2014, ISBN: 978-1-4822-2325-5, pp:7-9, 13, 14, 102-105.

VI. Annex

A. Shape of the reed



	S_0	S_1	S_2
x	Y=0	Y=4 mm	Y=6 mm
0	0,074	0,08	0,042
5	0,343	0,293	0,197
10	0,648	0,542	0,397
15	1,047	0,847	0,571
20	1,451	1,735	0,745
25	1,926	1,527	1,078
30	2,540	2,084	1,589
35	3,351	2,817	2,256

$$vamp\,(x,y) = p_0\,(x) + p_1\,(x)\cdot y^2 + p_0(x)\cdot y^4$$
$$p_0(x) = s_0(x)$$
$$p_1(x) = [-65\cdot s_0(x) + 81\cdot s_1(x) - 16\cdot s_2(x)]/720$$
$$p_2(x) = [5\cdot s_0 - 9\cdot s_1(x) + 4\cdot s_2(x)]/2880$$

$$heel(y) = -14'1 + \sqrt{17{,}4^2 - y^2}$$

$$contour(x)\begin{cases} 0 & x < 0\ or\ \ x \geq 67{,}5 \\ \sqrt{(24'4 - x)\cdot x} & x < 1{,}13196 \\ 4'08044 + \sqrt{-5'31 + 6'8\cdot x - x^2} & x > 2{,}94661 \\ \dfrac{263}{40} - \dfrac{11}{900}\cdot x & x < 67{,}5 \end{cases}$$

$$thickness(x,y)\begin{cases} \min\ [heel(y)\,, vamp(x,y)] & Abs(y) < contour(x) \\ 0 & Otherwise \end{cases}$$

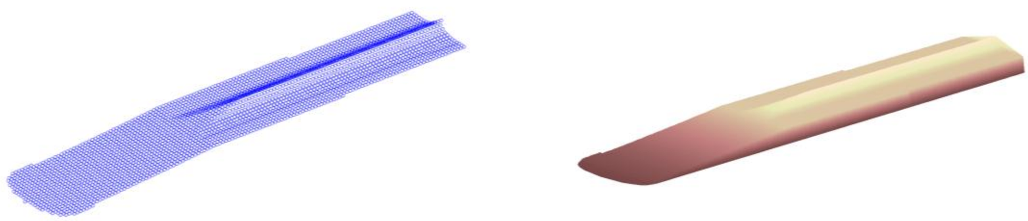


Fig. 56. Parametric reed representation with MATLAB:with points and with surface.

B. Modal Analysis

The equations of motion for an acoustic or structural system can be written as

$$(-\omega^2[\mathrm{M}] + jw[\mathrm{C}] + [\mathrm{K}])\{p\} = \{f\} \tag{26}$$

where [M] is the mass matrix, [C] is the damping matrix, [K] is the stiffness matrix, $\{p\}$ is the vector of nodal pressures for an acoustic system or displacements for a structural system, and $\{f\}$ is the acoustic or structural load applied to the system. For a basic modal analysis, it is assumed that there is no damping and no applied loads, so the damping matrix [C] and the load

vector $\{f\}$ are removed from Equation (1.1), leaving [4, Eq. (17.46)]

$$(-\omega^2[\mathrm{M}] + [\mathrm{K}])\{p\} = \{0\} \tag{27}$$

For an undamped system, the free pressure oscillations are assumed to be harmonic of the form

$$\{p\} = \{\phi\}_n \cos \omega_n t \tag{28}$$

where $\{\phi\}_n$ is the eigenvector of pressures of the nth natural frequency, ω_n is the natural circular frequency (radians/s), t is time. Substitution of Equation (1.3) into (1.2) gives

$$(-\omega_n{}^2[\mathrm{M}] + [\mathrm{K}])\{\phi\}_n = \{0\} \tag{29}$$

The trivial solution is $\{\phi\}_n = 0$. The next series of solutions is where the determinant equates to zero and is written as [4, Eq. (17-49)]

$$|[\mathrm{K}] - \omega_n{}^2[\mathrm{M}]| = 0 \tag{30}$$

which is a standard eigenvalue problem and is solved to find the natural frequencies (eigenvalues) ω_n and mode shapes (eigenvectors) $\{\phi\}_n$.. ANSYS will list results of the natural frequencies f_n in Hertz, rather than circular frequency in radians/s, where

$$f_n = \frac{\omega_n}{2\pi} \tag{31}$$

C. Harmonic Analysis

The harmonic response of a system can be calculated using two methods: full and modal summation (or superposition). The full method involves forming the mass [M], damping [C], and stiffness [K] matrices and the loading vector $\{f\}$ of the dynamic equations of motion, combining the matrices, then inverting the combined matrix and multiplying it with the load vector to calculate the nodal displacements fug, as follows [6]:

$$\begin{gathered}
[\mathrm{M}]\{\ddot{u}\} + [\mathrm{C}]\{\dot{u}\} + [\mathrm{K}]\{u\} = \{f\} \\
-\omega^2[\mathrm{M}]\{u\} + \mathrm{j}\omega[\mathrm{C}]\{u\} + [\mathrm{K}]\{u\} = \{f\} \\
(-\omega^2[\mathrm{M}] + jw[\mathrm{C}] + [\mathrm{K}])\{u\} = \{f\} \\
\{u\} = (-\omega^2[\mathrm{M}] + jw[\mathrm{C}] + [\mathrm{K}])^{-1}\{f\}
\end{gathered} \tag{32}$$

The modal summation method involves the calculation of the mode shapes of a structural or acoustic system, and determining what portion of each mode, called the modal participation factors P_n, contributes to the overall response.